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The motion of Saturn coorbital satellites in the restricted three-body problem

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Abstract. This paper provides a description of the motion of Saturn coorbital satellites Janus and Epimetheus by means of horseshoe periodic orbits in the framework of the planar restricted three-body problem for the mass parameter $\mu = 3.5 \times 10^{-9}$. The mechanism of existence of such orbits for any value of $\mu > 0$ and the Jacobi constant C close to $C(L_3)$, L_3 being an adequate collinear equilibrium point, is analyzed from two different points of view and a systematic way to compute the horseshoe periodic orbits is also described.

Key words. celestial mechanics – planets and satellites – solar system

1. Introduction

In 1981 the successful Voyager flights to Saturn confirmed the existence of two small satellites of Saturn, Janus (1908S1) and Epimetheus (1980S3), and provided an estimate of their masses as well as their orbital elements. These coorbital satellites turned out to be librating in horseshoe orbits, in an adequate rotating system. As their semimajor axes are only 50 km apart, they can approach within 15 000 km, but when they are close to each other, their mutual gravitational interaction prevents a collision and switches their orbits. Several authors have dealt with the coorbital motion in the framework of the planar three-body problem: up-to-date numerical values and a detailed history of the discovery is given in Aksnes (1985), Dermott et al. (1980) considered horseshoe orbits to account for the location of narrow rings of Saturn (and other planets), Dermott & Murray (1981a, 1981b) gave a description of the coorbital motion of 1980S1 and 1980S3 based on a combination of numerical integration and perturbation theory; they studied first the case where the mass of the third body was negligible, and generalized some of the results to include the case where the third body had sufficient mass to affect the other satellite (see also Murray & Dermott 1999). Later on, Yoder et al. (1983) derived a simple analytic approximation to the motion of all Trojan-like asteroids and applied it to 1980S1 and 1980S3. Rabe (1961) found one horseshoe periodic orbit in the restricted three-body problem for the particular value of the mass parameter $\mu = \mu_{\rm SJ}$, corresponding to the Sun-Jupiter system, and Taylor (1981) also computed segments of several families of horseshoe periodic orbits for $\mu_{\rm SJ}$. On the other hand, other mathematical theories have been developed: Garfinkel (1977, 1978, 1980) constructed a formal long-periodic solution in the restricted three-body problem, assuming the mass parameter small enough, and such solution embraced in particular horseshaped orbits.

Waldvogel and Spirig studied the problem of Saturn's coorbital satellites by means of a singular-perturbation approach, that is, the motion is initially described by an *outer* and an *inner approximation*, valid when the satellites are far apart or close together, respectively; the complete description of the motion requires the matching of both approximations (see Spirig & Waldvogel 1985; Waldvogel & Spirig 1988). In a similar way, Petit & Hénon (1986) dealt with satellite encounters in the framework of Hill's problem from analytical and numerical points of view. Finally, Cors & Hall (2001) approach this problem analytically by introducing small parameters into the usual three-body equations, truncating higher order terms and deriving dynamical information from the resulting equations.

Our approach deals with the existence of families of horseshoe periodic orbits in the *planar circular restricted three-body problem* (RTBP). More concretely, the aim of this paper is twofold: (i) first of all, we consider the RTBP where the primaries are Saturn and Janus, that is, with a mass parameter $\mu = 3.5 \times 10^{-9}$ (according to Yoder et al. 1989; Nicholson et al. 1992), and Epimetheus (the smaller satellite) is the third body of infinitesimal mass.

We show that the motion of Saturn coorbital satellites is closely related to some periodic orbits of this RTBP problem. We compute new families of stable horseshoe periodic orbits which approach the actual

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