Nonlinear Analysis: Real World Applications 14 (2013) 2002–2012

Contents lists available at SciVerse ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

On the existence and uniqueness of limit cycles in planar continuous piecewise linear systems without symmetry

Jaume Llibre [a], Manuel Ordóñez [b], Enrique Ponce [b,*]

[a] Departament de Matemàtiques, Facultat de Ciencies, Universitat Autònoma de Barcelona, Bellatera, 08193 Barcelona, Spain

[b] Departamento de Matemática Aplicada, Escuela Técnica Superior de Ingeniería, Avda. de los Descubrimientos, 41092 Sevilla, Spain

HIGHLIGHTS

- Non-symmetric planar continuous piecewise-linear differential systems are studied.
- Some results about the existence and uniqueness of their limit cycles are given.
- For systems with three linear zones and no symmetries new results are obtained.
- For systems with two linear zones a shorter proof of known results is achieved.
- The application to the McKean model of a single neuron activity is described.

ARTICLE INFO

Article history:
Received 13 May 2012
Accepted 10 February 2013

Keywords:
Piecewise linear systems
Liénard equation
Limit cycles

ABSTRACT

Some techniques to show the existence and uniqueness of limit cycles, typically stated for smooth vector fields, are extended to continuous piecewise-linear differential systems.

New results are obtained for systems with three linearity zones without symmetry and having one equilibrium point in the central region. We also revisit the case of systems with only two linear zones giving shorter proofs of known results.

A relevant application to the McKean piecewise linear model of a single neuron activity is included.

1. Introduction and statement of main results

For planar differential systems, the analysis of the possible existence of limit cycles and their uniqueness is a problem which has attracted the interest of many works in the past. For smooth systems, good classical references in the field are the books [1,2]. The restriction of this problem to polynomial differential equations is the well-known 16th Hilbert's problem [3]. Since Hilbert's problem turns out to be a strongly difficult one, Smale [4] has particularized it to Liénard differential systems in his list of problems for the present century.

For just continuous or even smooth Liénard systems there are many results on the non-existence, existence and uniqueness of limit cycles, see for instance [5–9,2]. Going beyond the smooth case, the first natural step is to allow non-smoothness while keeping the continuity, as it has been done in some recent works [10–13]. In a further step, other authors have considered a line of discontinuity in the vector field defining the planar system, see [14,15].

In this paper, we adapt some techniques from the smooth case to continuous piecewise linear differential systems, obtaining new results for systems without symmetry. We also revisit the case of systems with only two linear zones giving shorter proofs of known results.

* Corresponding author. Tel.: +34 954486172; fax: +34 954486165.
E-mail addresses: jllibre@mat.uab.cat (J. Llibre), mordonez@us.es (M. Ordóñez), eponcem@us.es, ponce.e@gmail.com (E. Ponce).

1468-1218/$ – see front matter
http://dx.doi.org/10.1016/j.nonrwa.2013.02.004