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On the Families of Periodic Orbits of the Sitnikov Problem*

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Abstract. The main goal of this paper is to study analytically the families of symmetric periodic orbits of the elliptic Sitnikov problem for all values of the eccentricity in the interval $[0, 1)$, providing qualitative and quantitative information on the bifurcation diagram of such families of periodic orbits. The basic tool for proving our results is the global continuation method of the zeros of a function depending on one parameter provided by Leray and Schauder and based in the Brouwer degree.

Key words. 3-body problem, Sitnikov problem, periodic orbits, global continuation

AMS subject classifications. Primary, 70F15; Secondary, 37N05

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1. Introduction.

The Sitnikov problem is a special case of restricted 3-body problems where the two primaries with equal masses are moving in a circular or an elliptic orbit of the 2-body problem, and the infinitesimal mass is moving on the straight line orthogonal to the plane of motion of the primaries which passes through their center of mass.

When the orbit described by the primaries is circular the Sitnikov problem is known as the *circular Sitnikov problem*. In 1907 Pavanini [33] expressed its solutions by means of Weierstrassian elliptic functions. Four years later MacMillan [26] expressed the solutions in terms of Jacobian elliptic functions (a detailed description of this work can be found in Stumpff [38]). Some other analytical expressions for the solutions of this problem can be found, for instance, in [39], [5], [42], [17], and [20]. The precise definition of the Sitnikov problem is given in section 2.

The *elliptic Sitnikov problem* is the case when the orbit describing the primaries is elliptic. This problem became important in 1960 when Sitnikov [37] used it to show, for the first time, the possibility of the existence of oscillatory motions in the 3-body problem. The existence of this kind of motion was predicted by Chazy [8], [9], [10] in 1922–1932, when he classified the final evolutions of the 3-body problem. Later on Alekseev [2], [3], [4] in 1968–1969 proved that, in the special case of the 3-body problem studied by Sitnikov, all of the possible combinations of final motions in the sense of Chazy were realized. Moser [32] in 1973 gave alternative proofs of the results of Alekseev which are simpler than those in [2], [3], [4]. Since then many other authors have studied the circular or elliptic Sitnikov problem—for instance, Llibre and Simó [24], Perdios and Markellos [34], Liu and Sun [21], Hagel [18], Hagel and Trenkler [19],

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