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Polynomial differential systems having a given Darbouxian first integral [☆]

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Abstract

The Darbouxian theory of integrability allows to determine when a polynomial differential system in $\mathbb{C}^2$ has a first integral of the kind $f_1^{\lambda_1} \dots f_p^{\lambda_p} \exp(g/h)$ where f_i , g and h are polynomials in $\mathbb{C}[x, y]$, and $\lambda_i \in \mathbb{C}$ for $i = 1, \dots, p$. The functions of this form are called Darbouxian functions. Here, we solve the inverse problem, i.e. we characterize the polynomial vector fields in $\mathbb{C}^2$ having a given Darbouxian function as a first integral.

On the other hand, using information about the degree of the invariant algebraic curves of a polynomial vector field, we improve the conditions for the existence of an integrating factor in the Darbouxian theory of integrability.

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1. Introduction and statement of the main results

By definition a *planar polynomial differential system* is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \quad \frac{dy}{dt} = \dot{y} = Q(x, y), \quad (1)$$

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