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Limit cycles bifurcating from a degenerate center

Jaume Llibre[a,*], Chara Pantazi[b]

[a] Departament de Matemàtiques, Universitat Autònoma de Barcelona, Edifici C, 08193 Bellaterra, Barcelona, Catalonia, Spain
[b] Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, (EPSEB), Av. Doctor Marañón, 44–50, 08028 Barcelona, Spain

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Abstract

We study the maximum number of limit cycles that can bifurcate from a degenerate center of a cubic homogeneous polynomial differential system. Using the averaging method of second order and perturbing inside the class of all cubic polynomial differential systems we prove that at most three limit cycles can bifurcate from the degenerate center. As far as we know this is the first time that a complete study up to second order in the small parameter of the perturbation is done for studying the limit cycles which bifurcate from the periodic orbits surrounding a degenerate center (a center whose linear part is identically zero) having neither a Hamiltonian first integral nor a rational one. This study needs many computations, which have been verified with the help of the algebraic manipulator Maple.

Keywords: Polynomial differential systems; Centers; Limit cycles; Averaging theory

1. Introduction

Hilbert in [16] asked for the maximum number of limit cycles which real polynomial differential systems in the plane of a given degree can have. This is actually the well known *16th Hilbert Problem*, see for example the surveys [17,18] and references therein. Recall that a *limit cycle* of a planar polynomial differential system is a periodic orbit of the system isolated in the set of all periodic orbits of the system.

Poincaré in [22] was the first to introduce the notion of a center for a vector field defined on the real plane. So according to Poincaré a *center* is a singular point surrounded by a neighborhood filled of periodic orbits with the unique exception of the singular point.

Consider the polynomial differential system

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

and as usually we denote by $^{\cdot} = d/dt$. Assume that system (1) has a center located at the origin. Then after a linear change of variables and a possible scaling of time system (1) can be written in one of the following forms

$$\text{(A)} \quad \begin{aligned} \dot{x} &= -y + F_1(x, y), \\ \dot{y} &= x + F_2(x, y), \end{aligned} \qquad \text{(B)} \quad \begin{aligned} \dot{x} &= y + F_1(x, y), \\ \dot{y} &= F_2(x, y), \end{aligned} \qquad \text{(C)} \quad \begin{aligned} \dot{x} &= F_1(x, y), \\ \dot{y} &= F_2(x, y), \end{aligned}$$

* Corresponding author.
E-mail addresses: jllibre@mat.uab.cat (J. Llibre), chara.pantazi@upc.edu (C. Pantazi).

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