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First integrals of local analytic differential systems

Jaume Llibre [a,*], Chara Pantazi [b], Sebastian Walcher [c]

[a] *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain*
[b] *Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, EPSEB, Av. Doctor Marañòn, 44-50, 08028 Barcelona, Spain*
[c] *Lehrstuhl A für Mathematik, RWTH Aachen, 52056 Aachen, Germany*

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Abstract

We investigate formal and analytic first integrals of local analytic ordinary differential equations near a stationary point. A natural approach is via the Poincaré–Dulac normal forms: If there exists a formal first integral for a system in normal form then it is also a first integral for the semisimple part of the linearization, which may be seen as "conserved" by the normal form. We discuss the maximal setting in which all such first integrals are conserved, and show that all first integrals are conserved for certain classes of reversible systems. Moreover we investigate the case of linearization with zero eigenvalues, and we consider a three-dimensional generalization of the quadratic Dulac–Frommer center problem.

MSC: primary 34C20, 34C45, 34M25

Keywords: Formal first integral; Normal form; Center; Reversible system

1. Introduction

The question whether a differential equation admits nonconstant first integrals in the neighborhood of a stationary point is of considerable interest for its qualitative analysis. Thus, for real planar systems the existence of a local analytic first integral in a neighborhood of a center with eigenvalues $\pm\beta i$ with real $\beta \neq 0$ was shown by Poincaré and Lyapunov, see [16,12,14].

* Corresponding author.
E-mail addresses: jllibre@mat.uab.cat (J. Llibre), Chara.Pantazi@upc.edu (C. Pantazi), walcher@mathA.rwth.aachen.de (S. Walcher).

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