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The Full Periodicity Kernel for σ Maps

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Let σ be the topological space formed by the points (x, y) of $\mathbb{R}^2$ such that either $x^2 + y^2 = 1$, or $0 \leqslant x \leqslant 2$ and $y = 1$. A σ map f is a continuous self-map of σ having the branching point $(0, 1)$ as a fixed point. We denote by $\operatorname{Per}(f)$ the set of periods of all periodic points of f, and by $\mathbb{N}$ the set of positive integers. We prove that if f is a σ map and $\{2, 3, 4, 5, 7\} \subseteq \operatorname{Per}(f)$, then $\operatorname{Per}(f) = \mathbb{N}$. Conversely, if $S \subseteq \mathbb{N}$ is a set such that for every σ map f, $S \subseteq \operatorname{Per}(f)$ implies $\operatorname{Per}(f) = \mathbb{N}$, then $\{2, 3, 4, 5, 7\} \subseteq S$.

1. INTRODUCTION

A connected finite regular graph (or *graph* for short) is a pair consisting of a connected Hausdorff space G and a finite subspace V (points of V are called *vertices*) such that the following conditions hold:

(1) $G \backslash V$ is the disjoint union of a finite number of open subsets $e_1, \ldots, e_k$ called *edges*. Each e_i is homeomorphic to an open interval of the real line.

(2) The boundary, $\operatorname{cl}(e_i) \backslash e_i$, of the edge e_i consists of two distinct vertices, and the pair $(\operatorname{cl}(e_i), e_i)$ is homeomorphic to the pair $([0, 1], (0, 1))$.

A vertex v which belongs to the boundary of at least three different edges is called a *branching point* of G.

A *G map f* is a continuous self-map of G having all branching points of G as fixed points. Note that all the continuous self-maps in a graph G

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