

PERIODS FOR TRANSVERSAL MAPS ON COMPACT MANIFOLDS WITH A GIVEN HOMOLOGY

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ABSTRACT. Let M be a compact C^1 differentiable manifold such that its rational homology is $H_j(M; \mathbb{Q}) \approx \mathbb{Q}$ if $j \in J \cup \{0\}$, and $H_j(M; \mathbb{Q}) \approx \{0\}$ otherwise. Here J is a subset of the set of natural numbers \mathbb{N} with cardinal 1, 2 or 3. A C^1 map $f : M \rightarrow M$ is called transversal if for all $m \in \mathbb{N}$ the graph of f^m intersects transversally the diagonal of $M \times M$ at each point (x, x) such that x is a fixed point of f^m . We study the set of periods of f by using the Lefschetz numbers for periodic points.

1. INTRODUCTION AND STATEMENT OF THE RESULTS

In dynamical systems it is often the case that differentiable topological information can be used to study qualitative and quantitative properties of the system. This paper deals with the problem of determining the periods of the periodic points of a class of C^1 self-maps given the homology class of the map. Similar problems have been studied in [6] for compact manifolds with homology $H_0(M; \mathbb{Q}) \approx \mathbb{Q}$, $H_1(M; \mathbb{Q}) \approx \mathbb{Q} \oplus \mathbb{Q}$, $H_j(M; \mathbb{Q}) \approx \{0\}$ for $j \neq 0, 1$. From other point of view periodic points for transversal maps have been studied by Franks in [4], [5] Matsuoka in [9], see also [2] and [8].

The preliminary notation and definitions which are necessary to state our main results are those of [6]. We include them here for completeness.

Let $f : X \rightarrow X$ be a continuous map. A *fixed point* of f is a point x of X such that $f(x) = x$. Denote the totality of fixed points by $\text{Fix}(f)$. The point $x \in X$ is *periodic with period m* if $x \in \text{Fix}(f^m)$ but $x \notin \text{Fix}(f^k)$ for all $k = 1, \dots, m - 1$. Let $\text{Per}(f)$ denote the set of all periods of periodic point of f .

Let M be a compact manifold of dimension n . A continuous map $f : M \rightarrow M$ induces endomorphisms $f_{*j} : H_j(M; \mathbb{Q}) \rightarrow H_j(M; \mathbb{Q})$ (for $j = 0, 1, \dots, n$) on the