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PERIODS FOR CONTINUOUS SELF-MAPS OF THE FIGURE-EIGHT SPACE

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Let **8** be the graph shaped like the number 8. This paper contains a characterization of all possible sets of periods for all continuous self-maps of **8** with the branching point fixed. We remark that this characterization is the first complete classification of the sets of periods for all continuous self-maps on a connected graph with negative Euler characteristic with fixed branching points.

Keywords: Graph; map; period; (periodic structure of graph maps).

1. Introduction and Statement of the Main Results

A *finite graph* (simply a *graph*) G is a Hausdorff space which has a finite subspace V (points of V are called *vertices*) such that $G\backslash V$ is the disjoint union of a finite number of open subsets $e_1, e_2, \ldots, e_k$ (called *edges*), each e_i is homeomorphic to an open interval of the real line, and one or two vertices are attached at the boundary of each edge.

The number of edges having a vertex as an endpoint (with the closed edges homeomorphic to a circle counted twice) will be called the *valence* of this vertex. A vertex of valence 1 is called an *endpoint* and a vertex of valence ≥ 3 is called a *branching point*. Given a graph G, $e(G)$ and $b(G)$ denote the number of its endpoints and branching points, respectively.

A *circuit* is a subset of G homeomorphic to a circle. The *Euler characteristic* $\chi(G)$ of G is $c - d$, where c and d are the number of connected components of G and the number of independent circuits of G taken as elements of the first homological group of G, respectively.

We call a continuous self-map f of G having all branching points of G as fixed points a *G-map.* A point x of G will be called *periodic* with respect to f of *period* n if n is the smallest positive integer such that $f^n(x) = x$. We denote by $\mathrm{Per}(f)$ the set of periods of all periodic points of f.

Let $\leq_2 = \leq_s$ be the Sarkovskii ordering defined on $\mathbb{N}_s = \mathbb{N} \cup \{2^\infty\}$. See [Alsedà *et al.*, 2000].