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PERIODIC ORBITS NEAR A HETEROCLINIC LOOP FORMED BY ONE-DIMENSIONAL ORBIT AND A TWO-DIMENSIONAL MANIFOLD: APPLICATION TO THE CHARGED COLLINEAR THREE-BODY PROBLEM

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This paper is devoted to the study of a type of differential systems which appear usually in the study of the Hamiltonian systems with two degrees of freedom. We prove the existence of infinitely many periodic orbits on each negative energy level. All these periodic orbits pass near to the total collision. Finally we apply these results to study the existence of periodic orbits in the charged collinear three-body problem.

Keywords: Heteroclinic loop; periodic orbits; charged 3-body problem.

1. Introduction

In this paper we deal with differential systems of the form

$$\begin{aligned} \dot{r} &= rv, \quad \dot{v} = \frac{v^2}{2} + u^2 - V(s), \\ \dot{s} &= u, \quad \dot{u} = -\frac{1}{2}vu + V'(s), \end{aligned} \tag{1}$$

where the dot denotes the derivative with respect to t, $V : (a, b) \to \mathbb{R}$ is a real function and $V'(s)$ means derivation with respect to the variable s. System (1) has the first integral

$$H = \frac{1}{r}\left[\frac{1}{2}(u^2 + v^2) - V(s)\right].$$

As we can see in [Devaney, 1981], these type of systems appear usually in the study of many Hamiltonian systems with two degrees of freedom. For example, are the study of the collinear three-body problem [McGehee, 1974], the charged rhomboidal four-body problem [Corbera *et al.*, 2005], the anisotropic problem [Casasayas & Llibre, 1984], etc.

System (1) has an invariant manifold, called the total *collision manifold*

$$\Lambda = \{(r, v, s, u) : r = 0,\ v^2 + u^2 = 2V(s),\ s \in (a, b)\}.$$

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