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Periodic solutions of a galactic potential

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ABSTRACT

We study analytically the periodic solutions of a Hamiltonian in $\mathbb{R}^6$ given by the kinetic energy plus a galactic potential, using averaging theory of first order. The model perturbs a harmonic oscillator, and has been extensively used in order to describe local motion in galaxies near an equilibrium point.

1. Introduction and statement of the main results

We consider the Hamiltonian

$$H = H(x,y,z,p_x,p_y,p_z) = \frac{1}{2}\left(p_x^2 + p_y^2 + p_z^2\right) + V(x,y,z),$$

where the potential $V = V(x,y,z)$ is given by

$$V = \frac{\omega^2}{2}(x^2+y^2+z^2) - \varepsilon\big(a(x^4+y^4+z^4) + 2b(x^2y^2+x^2z^2+y^2z^2)\big),$$

where ε is a small parameter (the perturbation strength), and a and b are parameters. Here ω is the common unperturbed frequency of the oscillations along the x, y and z axis. The potential V is a 3-dimensional perturbed harmonic oscillator and describes local motion in the central parts of a galaxy. It is called galactic potential. Such local 3-dimensional potentials appear after the expansion of global galactic potentials in a Taylor series near a stable equilibrium point and have been used by many authors in order to describe local motion in galaxies, see for instance Deprit and Elipe [3], Caranicolas [2], Elipe and Deprit [4], Elipe [5], Arribas et al. [1], Zotos [7–10], Zotos and Caranicolas [11], Zotos and Carpintero [12], ...

It is not restrictive, rescaling the variables (x,y,z) if necessary, to take $\omega = 1$. In short we shall study the periodic solutions of the Hamiltonian system

$$\begin{aligned}
\dot{x} &= p_x,\\
\dot{p}_x &= -x + \varepsilon(4ax^3 + 2b(2xy^2 + 2xz^2)),\\
\dot{y} &= p_y,\\
\dot{p}_y &= -y + \varepsilon(4ay^3 + 2b(2x^2y + 2yz^2)),\\
\dot{z} &= p_z,\\
\dot{p}_z &= -z + \varepsilon(4az^3 + 2b(2x^2z + 2y^2z)),
\end{aligned} \tag{1}$$

with Hamiltonian

$$H = \frac{1}{2}\left(p_x^2+p_y^2+p_z^2\right) + \frac{1}{2}(x^2+y^2+z^2) - \varepsilon\big(a(x^4+y^4+z^4) + 2b(x^2y^2+x^2z^2+y^2z^2)\big). \tag{2}$$

As far as we know there are no rigorous analytic studies of the existence of periodic solutions for the Hamiltonian system (1). The only studies in this direction are numerical or pseudo-numerical, (see [2] and the references therein). Here we shall study the periodic solutions of system (1) analytically by using the averaging theory, see for more details Section 2. In short, the averaging method, is the procedure of replacing a vector field by its average (over time or an angular variable) with the goal to obtain asymptotic approximations to the original system and to obtain periodic solutions.

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