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Transversal Homoclinic Orbits in the Collinear Restricted Three-Body Problem

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Summary. Using the perturbation method of Melnikov, we prove in a simple way the existence of transversal homoclinic points in the collinear restricted three-body problem. As a consequence we can embed a Bernoulli shift on a suitable cross section of the flow, showing easily that this problem possesses chaotic dynamics.

AMS Subject Classification. 70F10, 70H05

Key words. Melnikov method, homoclinic loop, McGehee coordinates

1. Introduction

Oscillatory motions in the three-body problem correspond to orbits whose behavior are unpredictable for long time intervals. Sitnikov [13] in 1963 was the first in providing a model of the restricted three-body problem which probably exhibits oscillatory motion. In 1968 Alekseev [1] proved the existence of such motion in the model proposed by Sitnikov. In 1973 Moser [11] provided an easier proof of this motion.

Later on, in 1980, Llibre and Simó [7] studied the restricted collinear three-body problem. They proved the existence of transversal homoclinic orbits in this problem, which allowed them to show the existence of the Smale horseshoe, and consequently the existence of chaotic motion in that model. In particular, their results show the existence of oscillatory motion in this different restricted three-body problem. In their proof the authors use variational calculus along a special parabolic orbit, and after difficult and tedious computations they get the result.