DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS SERIES B
Volume **19**, Number **6**, August **2014**

doi:10.3934/dcdsb.2014.19.1731

pp. **1731–1736**

ZERO-HOPF BIFURCATION FOR A CLASS OF LORENZ-TYPE SYSTEMS

Jaume Llibre

Departament de Matemátiques, Universitat Autónoma de Barcelona
08193 Bellaterra, Barcelona, Catalonia, Spain

Ernesto Pérez-Chavela

Departamento de Matemáticas, UAM-Iztapalapa
Av. Rafael Atlixco 186, México, DF 09340, Mexico

(Communicated by Shigui Ruan)

Abstract. In this paper we apply the averaging theory to a class of three-dimensional autonomous quadratic polynomial differential systems of Lorenz-type, to show the existence of limit cycles bifurcating from a degenerate zero-Hopf equilibrium.

1. **Introduction.** A generic Hopf bifurcation is a local bifurcation where a limit cycle bifurcates from an equilibrium point when one pair of complex eigenvalues cross the imaginary axis. There are many others non-generic or degenerate Hopf bifurcations. One of them is called the zero-Hopf bifurcation. *A zero-Hopf equilibrium* is an equilibrium point of a three-dimensional autonomous differential system, which has a zero eigenvalue and a pair of purely imaginary eigenvalues. Many different kind of bifurcations can take place in a zero-Hopf equilibrium. When a periodic orbit bifurcates from a zero-Hopf equilibrium we will say that the system exhibits a *zero-Hopf bifurcation*. It has been studied by many authors among them Guckenheimer, Holmes, Scheurle, Han, Kuznetsov, Llibre and Zhang in [2, 3, 4, 6, 9].

In [5] the authors study three-dimensional autonomous quadratic polynomial differential systems having the four basic qualitative properties of the classical Lorenz system, that is:

- Symmetry with respect to the z-axis.
- Dissipation and existence of an attractor: As $t \to \infty$ all solutions of the differential system approach to a set of zero volume, i.e.,
$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} < 0.$$
- Pitchfork bifurcation. For certain choice of the parameters, the origin is an equilibrium point and from it bifurcate other two new equilibria. The equilibrium point at the origin changes its stability at the bifurcating point.
- The system can also exhibit Hopf bifurcations, homoclinic or heteroclinic orbits, and chaotic attractor.

2010 *Mathematics Subject Classification.* 37G15, 34C29.

Key words and phrases. Three-dimensional differential systems, zero-Hopf equilibrium, limit cycles, Lorenz-type system.