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PHASE PORTRAITS OF BERNOULLI QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS

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Abstract. In this article we study a new class of quadratic polynomial differential systems. We classify all global phase portraits in the Poincaré disk of Bernoulli quadratic polynomial differential systems in $\mathbb{R}^2$.

1. Introduction

Quadratic polynomial differential systems appear frequently in many areas of applied mathematics, electrical circuits, astrophysics, in population dynamics, chemistry, neural networks, laser physics, hydrodynamics, etc. Although these differential systems are the simplest nonlinear polynomial systems, they are also important as a basic testing ground for the general theory of the nonlinear differential systems.

There are more than a thousand papers written on the quadratic polynomial differential systems. For example there is a bibliography of some of these compiled by Reyn which has 426 items plus 55 preprints and 10 Reports published in TUDelft series of reports in 1989. See the books of Ye Yanqian et al. [24], Reyn [20], and Artés, Llibre, Schlomiuk and Vulpe [2] dedicated to the quadratic polynomial differential systems. See also the classical surveys on these systems by Coppel [6], and Chicone and Jinghuang [5].

Consider the differential equation

$$\frac{dy}{dx} = A(x)y^k + B(x)y, \tag{1.1}$$

with $k \in \mathbb{R} \setminus \{0, 1\}$ and A, B non zero real functions. This differential equation is called Bernoulli differential equation. Associated to the Bernoulli differential equation we can define the Bernoulli differential system given by

$$\begin{aligned} \dot{x} &= p(x), \\ \dot{y} &= a(x)y^k + b(x)y. \end{aligned} \tag{1.2}$$

Note that this system is equivalently equation (1.1).

In this article we consider Bernoulli polynomial differential system of degree 2 in $\mathbb{R}^2$, i.e. $p(x)$ is a polynomial with degree at most 2, $k = 2$, $a(x)$ is a constant non zero, and $b(x)$ is a non zero polynomial of degree at most 1 (otherwise the

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