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PHASE PORTRAITS OF A NEW CLASS OF INTEGRABLE QUADRATIC VECTOR FIELDS

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Abstract. We classify the phase portraits of all real quadratic polynomial vector fields having one real invariant algebraic curve of degree 3 such that its complex irreducible factors satisfy some generic assumptions. These vector fields have a first integral.
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1 Introduction

We consider a system of differential equations in $\mathbb{R}^2$ defined by

$$x' = P(x,y), \qquad y' = Q(x,y), \tag{1}$$

where P and Q are polynomials. We say that $m = \max\{\deg P, \deg Q\}$ is the *degree* of system (1). We can associate to system (1) the vector field $X = P\dfrac{\partial}{\partial x} + Q\dfrac{\partial}{\partial y}$.

In this work, we say that system (1) is *integrable* on an open subset $U \subset \mathbb{R}^2$ if the Lebesgue measure of $\mathbb{R}^2 \backslash U$ is zero and there exists a nonconstant analytical function $H : U \longrightarrow \mathbb{R}$ that is constant on each integral curve of (1) contained in U. The function H is called a *first integral* of (1) on U. It is easy to see that H is a first integral of (1) on U if and only if $XH \equiv 0$ on U.

If U satisfies the above assumptions and R is a nonconstant real analytical function on U such that $\dfrac{\partial(RP)}{\partial x} = -\dfrac{\partial(RQ)}{\partial y}$ then we say that R is an integrating factor of (1). Using an integrating factor it is possible to compute a first integral.

In what follows $\mathbf{F}$ will denote either the real field $\mathbb{R}$ or the complex field $\mathbb{C}$, and $\mathbf{F}[x,y]$ will denote the ring of all polynomials in the variables x and y with coefficients in $\mathbf{F}$. An *invariant algebraic curve* of system (1) is a curve $f(x,y) = 0$ where $f \in \mathbf{F}[x,y]$ such that there is some polynomial $K \in \mathbf{F}[x,y]$ that verifies $Xf = Kf$. The polynomial K is called the *cofactor* of the invariant algebraic curve $f = 0$.