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STRUCTURAL STABILITY OF PLANAR SEMI–HOMOGENEOUS POLYNOMIAL VECTOR FIELDS APPLICATIONS TO CRITICAL POINTS AND TO INFINITY

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Abstract. Recently, in [9] we characterized the set of planar homogeneous vector fields that are structurally stable and we obtained the exact number of the topological equivalence classes. Furthermore, we gave a first extension of the Hartman-Grobman Theorem for planar vector fields. In this paper we study the structural stability in the set $H_{m,n}$ of planar semi-homogeneous vector fields $X = (P_m, Q_n)$, where P_m and Q_n are homogeneous polynomial of degree m and n respectively, and $0 < m < n$. Unlike the planar homogeneous vector fields, the semi-homogeneous ones can have limit cycles, which prevents to characterize completely those planar semi-homogeneous vector fields that are structurally stable. Thus, in the first part of this paper we will study the local structural stability at the origin and at infinity for the vector fields in $H_{m,n}$. As a consequence of these local results, we will complete the extension of the Hartman-Grobman Theorem to the nonlinear planar vector fields. In the second half of this paper we define a subset $\Delta_{m,n}$ that is dense in $H_{m,n}$ and whose elements are structurally stable. We prove that there exist vector fields in $\Delta_{m,n}$ that have at least $\frac{m+n}{2}$ hyperbolic limit cycles.

1. **Introduction and Main Results.** We denote by $H_{m,n}$ the set of planar semi-homogeneous vector fields $X = (P_m,\ Q_n)$, where P_m and Q_n are homogeneous polynomial of degree m and n respectively, and $0 < m < n$. As far as we know few research has been down about semi–homogeneous vector fields. In 1966, Lyapunov proposed special coordinates for the study of the quasi-homogeneous vector fields that contain the semi–homogeneous ones. By using these coordinates Cima, Gasull and Mañosas in [4] have obtained $(n + m)/2$ as a lower bound for the number of limit cycles of $H_{m,n}$. Recently, Cairó and Llibre in [2] and [3] obtained the global phase portraits modulo limit cycles of $H_{1,2}$ and $H_{1,3}$ respectively.

The system of differential equations associated to $X \in H_{n,m}$ is

$$x' = \frac{dx}{dt} = P_m(x, y)\ , \qquad y' = \frac{dy}{dt} = Q_n(x, y). \tag{1}$$

In order to study the phase portraits in a neighborhood of the origin we use the change of variables $(x, y) \to (\overline{x}, \overline{y}, \epsilon)$ where $x = \epsilon\overline{x}$, $y = \epsilon\overline{y}$, $\overline{x}^2 + \overline{y}^2 = 1$ and

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