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Averaging analysis of a perturbated quadratic center☆

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1. Introduction and the main result

The main open problem in the qualitative theory of real planar differential systems is the determination and distribution of their limit cycles. A classical way to produce limit cycles is by perturbing a system which has a center, in such a way that limit cycles bifurcate in the perturbed system from some of the periodic orbits of the center for the unperturbed system (see for instance [6]). It is well known (see for example [2]) that perturbing the linear center $x' = -y,\ y' = x$ by arbitrary polynomials p and q of degree n (i.e. $x' = -y + \varepsilon p(x, y),\ y' = x + \varepsilon q(x, y)$), we can obtain up to first order in ε at most $[(n-1)/2]$ bifurcated limit cycles, where [ ] denotes the integer part function. Also, it is known that perturbing the quadratic center $x' = -y(1+x),\ y' = x(1+x)$ (note that essentially it is the linear center with a straight line of singular points) inside the quadratic systems we obtain two bifurcated limit cycles (see [5]), instead of $\left[\frac{1}{2}\right] = 0$. In this paper we shall prove that if we perturb the above quadratic system inside the polynomial systems of degree n we can obtain up to first order in ε at most n limit cycles.

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