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INVARIANT CIRCLES FOR HOMOGENEOUS POLYNOMIAL VECTOR FIELDS ON THE 2–DIMENSIONAL SPHERE

JAUME LLIBRE – CLAUDIO PESSOA

Let X be a homogeneous polynomial vector field of degree $n \geq 3$ on $\mathbb{S}^2$ having finitely many invariant circles. Then, for such a vector field X we find upper bounds for the number of invariant circles, invariant great circles, invariant circles intersecting at a same point and parallel circles with the same director vector. We give examples of homogeneous polynomial vector fields of degree 3 on $\mathbb{S}^2$ having finitely many invariant circles which are not great circles, which are limit cycles, but are not great circles and invariant great circles that are limit cycles. Moreover, for the case $n = 3$ we determine the maximum number of parallel invariant circles with the same director vector.

1. Introduction.

A *polynomial vector field X in* $\mathbb{R}^3$ is a vector field of the form

$$X = P(x, y, z)\frac{\partial}{\partial x} + Q(x, y, z)\frac{\partial}{\partial y} + R(x, y, z)\frac{\partial}{\partial z}, \tag{1.1}$$

where P, Q, R are polynomials in the variables x, y and z with real coefficients. We denote $n = \max\{\deg P, \deg Q, \deg R\}$ the *degree* of the polynomial vector field X. In what follows X will denote the above polynomial vector field.

Let $\mathbb{S}^2$ be the 2–dimensional sphere $\{(x, y, z) \in \mathbb{R}^3 \ : x^2 + y^2 + z^2 = 1\}$. A *polynomial vector field X on* $\mathbb{S}^2$ is a polynomial vector field in $\mathbb{R}^3$ such that restricted to the sphere $\mathbb{S}^2$ defines a vector field on $\mathbb{S}^2$; i.e. it must satisfy the following equality

$$xP(x, y, z) + yQ(x, y, z) + zR(x, y, z) = 0, \tag{1.2}$$

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