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Homogeneous Polynomial Vector Fields of Degree 2 on the 2–Dimensional Sphere

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1. Introduction and statement of the main results

A *polynomial vector field* X *in* $\mathbb{R}^3$ is a vector field of the form

$$X = P(x,y,z)\frac{\partial}{\partial x} + Q(x,y,z)\frac{\partial}{\partial y} + R(x,y,z)\frac{\partial}{\partial z},$$

where P, Q, R are polynomials in the variables x, y and z with real coefficients. We denote $m = \max\{\deg P, \deg Q, \deg R\}$ the *degree* of the polynomial vector field X. In what follows X will denote the above polynomial vector field.

Let $\mathbb{S}^2$ be the 2–dimensional sphere $\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$. A *polynomial vector field* X *on* $\mathbb{S}^2$ is a polynomial vector field in $\mathbb{R}^3$ such that restricted to the sphere $\mathbb{S}^2$ defines a vector field on $\mathbb{S}^2$; i.e. it must satisfy the following equality

$$xP(x,y,z) + yQ(x,y,z) + zR(x,y,z) = 0, \tag{1}$$

for all points (x,y,z) of the sphere $\mathbb{S}^2$.

Let $f \in \mathbb{R}[x,y,z]$, where $\mathbb{R}[x,y,z]$ denotes the ring of all polynomials in the variables x, y and z with real coefficients. The algebraic surface $f = 0$ is an *invariant algebraic surface* of the polynomial vector field X if for some polynomial $K \in \mathbb{R}[x,y,z]$ we have

$$Xf = P\frac{\partial f}{\partial x} + Q\frac{\partial f}{\partial y} + R\frac{\partial f}{\partial z} = Kf.$$