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HOPF BIFURCATION FROM INFINITY FOR PLANAR CONTROL SYSTEMS

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Abstract

Symmetric piecewise linear bi-dimensional systems are very common in control engineering. They constitute a class of non-differentiable vector fields for which classical Hopf bifurcation theorems are not applicable. For such systems, sufficient and necessary conditions for bifurcation of a limit cycle from the periodic orbit at infinity are given.

1. Introduction and statement of main results

In this paper we are concerned with the appearance of one limit cycle from infinity for symmetric piecewise linear bi-dimensional systems. This phenomenon can be considered as a kind of generalized Hopf bifurcation from the infinity.

The systems under study are of great importance in direct control theory [**1**], [**2**], being very common in control engineering as they include the case where the nonlinearities involved are of saturation type. They constitute a class of non-differentiable vector fields for which classical Hopf bifurcation theorems are not applicable so that specific techniques are needed in their analysis.

Thus, we consider differential systems of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \psi(\mathbf{c}^T\mathbf{x})\mathbf{b}, \tag{1}$$

where $\mathbf{A}$ is a 2×2 real matrix and $\mathbf{x}, \mathbf{b}, \mathbf{c}$ belong to $\mathbf{R}^2$. Here the dot denotes derivatives with respect to the variable s. The nonlinearity of these systems results from the presence of the *characteristic function* ψ. A common assumption in control theory is to consider odd piecewise linear characteristic functions of the form

$$\psi(\sigma) = \begin{cases} k_2\sigma - (k_1 - k_2)w & \text{if } \sigma \leq -w, \\ k_1\sigma & \text{if } -w < \sigma < w, \\ k_2\sigma + (k_1 - k_2)w & \text{if } w \leq \sigma. \end{cases} \tag{2}$$