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Bifurcation of a periodic orbit from infinity in planar piecewise linear vector fields

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1. Introduction and statement of the main results

The study of appearance of limit cycles by varying the coefficients of a planar vector field goes back to Poincaré. The simplest case (Hopf bifurcation) describes the simultaneous appearance of one or several limit cycles from a singular point. In this paper we are concerned with the appearance of one limit cycle from infinity. This bifurcation represents a form of generalized Hopf bifurcation from the infinity.

For differential systems in $\mathbf{R}^n$ *linearly dominated at infinity* depending on a parameter μ, Glover [5] and He [8] give sufficient conditions in order that a periodic orbit bifurcates from infinity at a critical value of the parameter, say $\mu = 0$. They do not provide any information about

(i) whether the bifurcating periodic orbit exists for $\mu < 0$ or $\mu > 0$ with $|\mu|$ sufficiently small,
(ii) the uniqueness of the bifurcating periodic orbit,
(iii) the stability of the bifurcating periodic orbit, and
(iv) an asymptotic estimate for the size of the bifurcating periodic orbit.

One of the main interests in this paper is the study of the bifurcation of a periodic orbit from infinity for differential systems of the form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \psi(\mathbf{c}^{\mathrm{T}}\mathbf{x})\mathbf{b}, \tag{1}$$

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