International Journal of Bifurcation and Chaos, Vol. 13, No. 4 (2003) 895–904

PIECEWISE LINEAR FEEDBACK SYSTEMS WITH ARBITRARY NUMBER OF LIMIT CYCLES

JAUME LLIBRE
Departament de Matemàtiques,
Universitat Autònoma de Barcelona,
08193 Bellaterra, Spain
jllibre@mat.uab.es

ENRIQUE PONCE
Departamento de Matemática Aplicada II,
E. S. Ingenieros, Camino de los Descubrimientos,
41092 Sevilla, Spain
enrique@matinc.us.es

Received January 11, 2002; Revised February 28, 2002

Given an arbitrary positive integer n, it is shown that there exist planar piecewise linear differential systems with at least n limit cycles.

Keywords: Nonlinear oscillations; limit cycles; averaging method.

1. Introduction and Statement of the Main Results

There is the feeling that piecewise linear differential systems can present all the complex dynamics one can see in the nonlinear differential systems. For instance, limit cycles, homoclinic and heteroclinic orbits which are the main ingredients for the qualitative description of the phase portraits of planar differential systems are also present in the planar piecewise linear differential systems. In higher dimension, even strange attractors appear in the class of piecewise linear differential systems. Following this line of thought, in this work we show that planar piecewise linear feedback systems can present so many limit cycles as we want.

One of the main problems in the qualitative study of planar differential equations is to know the number and distribution of their limit cycles. As far as we know, for piecewise linear differential systems there are only analytical results about systems with a low number of limit cycles, see for instance [Llibre & Sotomayor, 1996; Freire *et al.*, 1999; Freire *et al.*, 2002; Teruel, 2000; Llibre & Ponce, 1999].

In this paper, we provide an analytical proof that a piecewise linear differential system, which can be thought as a feedback system in control theory, can have as many limit cycles as we want. More specifically, our main result is the following one.

Theorem 1. *Let $\varphi : \mathbb{R} \to \mathbb{R}$ be an odd piecewise linear periodic function of period 4 such that $\varphi(x) = x$ if $x \in [0, 1]$ and $\varphi(x) = -x + 2$ if $x \in [1, 2]$, see Fig. 1. For an arbitrary positive integer n the following piecewise linear feedback system*

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \varphi(x) \begin{pmatrix} \varepsilon \\ 0 \end{pmatrix} \tag{1}$$

has at least n hyperbolic limit cycles in the strip $|x| \le 2n + 2$ for $\varepsilon \ne 0$ sufficiently small.