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Algebraic determination of limit cycles in a family of three-dimensional piecewise linear differential systems[☆]

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ABSTRACT

We study a one-parameter family of symmetric piecewise linear differential systems in $\mathbb{R}^3$ which is relevant in control theory. The family, which has some intersection points with the adimensional family of Chua's circuits, exhibits more than one attractor even when the two matrices defining its dynamics in each zone are stable, in an apparent contradiction to the three-dimensional Kalman's conjecture.

For these systems we characterize algebraically their symmetric periodic orbits and obtain a partial view of the one-parameter unfolding of its triple-zero degeneracy. Having at our disposal exact information about periodic orbits of a family of nonlinear systems, which is rather unusual, the analysis allows us to assess the accuracy of the corresponding harmonic balance predictions. Also, it is shown that certain conditions in Kalman's conjecture can be violated without losing the global asymptotic stability of the origin.

1. Introduction and statement of the main results

In control theory (see [1–5]), an important class of differential systems is the Lur'e system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}\xi(t), \qquad \sigma(t) = \mathbf{c}^T\mathbf{x}(t), \tag{1}$$

where $\mathbf{A}$ is an $n \times n$ constant matrix, $\mathbf{b}, \mathbf{c} \in \mathbb{R}^n$, and the *input* $\xi(t) = \varphi(\sigma(t))$ is the feedback of the *output* σ through a nonlinear continuous function $\varphi : \mathbb{R} \to \mathbb{R}$. Typically $\varphi(0) = 0$, so the origin is an equilibrium point. One of the main problems in this theory is to characterize when the origin is a global attractor. Related to this problem there is *Kalman's conjecture* [6] which states that if $k_1 \le \varphi'(\sigma) \le k_2$ and the linear systems $\dot{\mathbf{x}}(t) = (\mathbf{A} + k\mathbf{b}\mathbf{c}^T)\mathbf{x}(t)$ have the origin as a global attractor for all $k \in [k_1, k_2]$, then system (1) has also the origin as a global attractor. Kalman's conjecture is true for dimension $n \le 3$ and fails for $n > 3$; see [7,8].

We restrict our attention to system (1) in $\mathbb{R}^3$, so that we can write $\mathbf{x}(t) = (x(t), y(t), z(t)) \in \mathbb{R}^3$, and without loss of generality we assume that $\mathbf{c} = (1, 0, 0)^T$. A common case in applications is when the nonlinear characteristic function is the saturation

$$\varphi(\sigma) = \sigma \quad \text{for } |\sigma| \le 1, \qquad \varphi(\sigma) = \operatorname{sgn}(\sigma) \quad \text{for } |\sigma| > 1, \tag{2}$$

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