On the fold-Hopf bifurcation for continuous piecewise linear differential systems with symmetry

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In this paper a partial unfolding for an analog to the fold-Hopf bifurcation in three-dimensional symmetric piecewise linear differential systems is obtained. A particular biparametric family of such systems is studied starting from a very degenerate configuration of nonhyperbolic periodic orbits and looking for the possible bifurcation of limit cycles. It is proved that four limit cycles can coexist after perturbation of the original configuration, and other two limit cycles are conjectured. It is shown that the described bifurcation scenario appears for appropriate values of parameters in the celebrated Chua's oscillator. © *2010 American Institute of Physics*. [doi:10.1063/1.3486073]

In this paper a partial unfolding for an analog to the fold-Hopf bifurcation in symmetric piecewise linear systems with three zones of linearity is obtained. The global vector field considered is continuous but nonsmooth. The dynamics at the origin undergoes the same degeneracy as the one in the fold-Hopf bifurcation for smooth systems, but we assume that the dynamics in the outer zones is governed by negative real eigenvalues. More precisely, we assume that the eigenvalues in the central zone are ε and $-\varepsilon \pm i$ and the eigenvalues in the external zones are $-\mu$, -2μ, and -3μ. Thus by moving the parameter ε we get a simultaneous crossing of three eigenvalues through the imaginary axis trying to emulate the so-called fold-Hopf bifurcation for differentiable systems. The particular choice of the external eigenvalues facilitates certain algebraic computations. This analysis should be considered as a first step in looking for a more general analysis to be done elsewhere. Here, for certain values of parameters, we prove in a rigorous way the coexistence of four limit cycles and characterize their stability. We conjecture the bifurcation of two additional limit cycles. Finally we show that this scenario appears in the Chua's circuit for appropriate values of the parameters.

I. INTRODUCTION AND PRELIMINARY RESULTS

The interest on the analysis of piecewise linear differential systems (simply piecewise linear systems in what follows) has increased in the past decades as modern engineering applications require the piecewise linear modeling of a wide range of problems in mechanics, power electronics, control theory, biology, and so on, see Ref. 1. On the one hand, piecewise linear systems are the natural extension of the linear ones in order to cope with nonlinear phenomena, so that they can reproduce much of the complex behavior observed in smooth nonlinear systems: multistability, self-sustained oscillations, hysteretic behavior, homoclinic and heteroclinic connections, and of course, chaotic behavior. On the other hand, piecewise linear systems turn out to be the most accurate models in some realistic applications of the quoted fields.[2]

Piecewise linear systems can be classified in two big classes depending on the continuity of the associated vector field. The discontinuous case will not be considered here although it is nowadays the subject of intense research, see again Ref. 1. In fact, there are still many unsolved issues in the continuous case, even for the seemingly simple problem of stability of the only equilibrium point, see Ref. 3. Apart from equilibria, it is very important to characterize the periodic orbits of such systems, since they constitute the next step in complexity for observed behavior in practice. To study the existence of periodic orbits for piecewise linear systems, we will follow a point of view which is typical in bifurcation theory, that is, we study degenerated situations and after parameter variations we look for the appearance of limit cycles.

Unfortunately the nonsmoothness of continuous piecewise linear systems requires that limit cycle bifurcations must be analyzed in a case-by-case approach for the different families of systems which are relevant in applications. Thus both planar and three-dimensional cases with and without symmetry have been considered in previous works.[4–7] In all the quoted cases the bifurcation giving rise to limit cycles is associated to the crossing, for one linear piece of the vector field, of one eigenvalue pair through the imaginary axis of the complex plane, in a similar way to what happens in smooth Poincaré–Andronov–Hopf bifurcations.

Here, going a step farther, we will consider a specific situation where such an eigenvalue pair crossing occurs simultaneously with the presence of one additional zero eigenvalue, what is reminiscent of the so-called *fold-Hopf bifurcation* (sometimes named *Hopf-zero bifurcation*) for differentiable systems, see Sec. 7.4 of Ref. 8 and Sec. 8.5 of

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