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On the existence and uniqueness of limit cycles in Liénard differential equations allowing discontinuities

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Abstract
In this paper we study the non-existence and the uniqueness of limit cycles for the Liénard differential equation of the form $x'' - f(x)x' + g(x) = 0$ where the functions f and g satisfy $xf(x) > 0$ and $xg(x) > 0$ for $x \neq 0$ but can be discontinuous at $x = 0$.

In particular, our results allow us to prove the non-existence of limit cycles under suitable assumptions, and also prove the existence and uniqueness of a limit cycle in a class of discontinuous Liénard systems which are relevant in engineering applications.

Mathematics Subject Classification: 58F21, 34C05, 58F14

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Piecewise smooth dynamical systems serve as models for a great variety of engineering devices and they deserve considerable attention, see for instance the recent book [1] and references therein. For instance, in modern nonlinear control techniques the lack of smoothness is sometimes enforced either by the consideration of hybrid systems or by the artificial introduction of discontinuities, see [23].

Even for low-dimensional continuous models the analysis of non-smooth systems is an intricate problem. Perhaps one of the most striking examples is the seemingly simplest case of continuous piecewise linear differential systems with only two regions separated by a hyperplane which contains the unique equilibrium point. Surprisingly enough, in dimension three or higher the stability of such an equilibrium has not yet been explicitly characterized, see [3].

