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Existence of piecewise linear differential systems with exactly n limit cycles for all $n \in \mathbb{N}$

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Abstract

In this paper, we prove that the piecewise linear differential system

$$\dot{x} = -y - \varepsilon\phi(x), \qquad \dot{y} = x$$

with $\varepsilon \neq 0$ and ϕ an odd piecewise linear periodic function of period 4, has exactly n limit cycles in the strip $|x| \leqslant 2(n+1)$. Consequently, there are piecewise linear differential systems having infinitely many limit cycles in the real plane. We also provide examples of piecewise linear differential systems having exactly n limit cycles for all $n \in \mathbb{N}$.

MSC: 34C05; 34C25; 37G15

Keywords: Piecewise linear control system; Limit cycles; Nonlinear oscillations

1. Introduction and statement of the main results

Piecewise linear differential systems appear extensively in control theory (see for instance, [1,2,5]). These systems have the same complicated dynamical phenomena than those in nonlinear differential systems, for instance, limit cycles [6,7,9,10], homoclinic and heteroclinic orbits [8] which are the main ingredients in qualitative description of

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