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Center problem for Λ–Ω differential systems

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Abstract

The Λ-Ω systems are the real planar polynomial differential equations of degree m

$$\dot{x} = -y(1+\Lambda) + x\Omega, \qquad \dot{y} = x(1+\Lambda) + y\Omega,$$

where $\Lambda = \Lambda(x, y)$ and $\Omega = \Omega(x, y)$ are polynomials of degree at most $m-1$ such that $\Lambda(0, 0) = \Omega(0, 0) = 0$. We study the center problem for these Λ-Ω systems. Any planar vector field with linear type center can be written as a Λ-Ω system if and only if the Poincaré-Liapunov first integral is of the form $F = \frac{1}{2}(x^2 + y^2)(1 + O(x, y))$. These kind of linear type centers are called weak centers, they contain the class of centers studied by Alwash and Lloyd [1], and also contain the uniform isochronous centers, and the holomorphic isochronous centers, but they do not coincide with the all class of isochronous centers.

The main objective of this paper is to study the center problem for two particular classes of Λ-Ω systems of degree m.

First if $\Lambda = \mu(a_2x - a_1y)$, and $\Omega = a_1x + a_2y + \Omega_{m-1}$, where μ, a_1, a_2 are constants and $\Omega_{m-1} = \Omega_{m-1}(x, y)$ is a homogenous polynomial of degree $m-1$, then we prove the following results.

(i) These Λ-Ω systems have a weak center at the origin if and only if $(\mu + m - 2)(a_1^2 + a_2^2) = 0$, and $\int_0^{2\pi} \Omega_{m-1}(\cos t, \sin t)dt = 0$;

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