Integrability of a class of N–dimensional Lotka–Volterra and Kolmogorov systems

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Abstract

We study the integrability of an N–dimensional differential Kolmogorov systems of the form

$$\dot{x}_j = x_j \left(a_j + \sum_{k=1}^{N} a_{jk} x_k \right) + x_j \Psi(x_1, \ldots, x_N), \quad j = 1, \ldots, N,$$

where a_j, and a_{jk} are constants for $j, k = 1, \ldots, N$ and $\Psi(x_1, \ldots, x_N)$ is a homogeneous polynomial of degree $n > 2$, with either one additional invariant hyperplane, or with one exponential factor. We also study the integrability of the N–dimensional classical Lotka-Volterra systems (when $\Psi(x_1, \ldots, x_N) = 0$). In particular we consider the integrability of the asymmetric May–Leonard systems.
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