



The 16th Hilbert problem restricted to circular algebraic limit cycles

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Abstract

We prove the following two results. First every planar polynomial vector field of degree S with S invariant circles is Darboux integrable without limit cycles. Second a planar polynomial vector field of degree S admits at most $S - 1$ invariant circles which are algebraic limit cycles. In particular we solve the 16th Hilbert problem restricted to algebraic limit cycles given by circles, because a planar polynomial vector field of degree S has at most $S - 1$ algebraic limit cycles given by circles, and this number is reached.

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