



Contents lists available at ScienceDirect

Journal of Differential Equations

www.elsevier.com/locate/jde



On the 16th Hilbert problem for algebraic limit cycles

Jaume Llibre^{a,*}, Rafael Ramírez^b, Natalia Sadovskaia^c

^a Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain

^b Departament d'Enginyeria Informàtica i Matemàtiques, Universitat Rovira i Virgili, Avinguda dels Països Catalans 26, 43007 Tarragona, Catalonia, Spain

^c Departament de Matemàtica Aplicada II, Universitat Politècnica de Catalunya, C. Pau Gargallo 5, 08028 Barcelona, Catalonia, Spain

ARTICLE INFO

Article history:

Received 2 April 2009

Revised 6 November 2009

Available online 3 December 2009

MSC:

34C05

34A34

34C14

Keywords:

Limit cycles

Algebraic limit cycles

Polynomial vector fields

16th Hilbert problem

ABSTRACT

For a polynomial planar vector field of degree $n \geq 2$ with generic invariant algebraic curves we show that the maximum number of algebraic limit cycles is $1 + (n-1)(n-2)/2$ when n is even, and $(n-1)(n-2)/2$ when n is odd. Furthermore, these upper bounds are reached.

© 2009 Elsevier Inc. All rights reserved.

1. Introduction and statement of the main results

A real (planar) polynomial differential system is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \quad \frac{dy}{dt} = \dot{y} = Q(x, y), \quad (1)$$

where P and Q are real polynomials in the variables x and y . In this work the dependent variables x and y , the independent variable t , and the coefficients of the polynomials P and Q are all real be-

* Corresponding author.

E-mail addresses: llibre@mat.uab.cat (J. Llibre), rafaelorando.ramirez@urv.cat (R. Ramírez), natalia.sadovskaia@upc.edu (N. Sadovskaia).