



On the 16th Hilbert problem for limit cycles on nonsingular algebraic curves

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ABSTRACT

We give an upper bound for the maximum number N of algebraic limit cycles that a planar polynomial vector field of degree n can exhibit if the vector field has exactly k nonsingular irreducible invariant algebraic curves. Additionally we provide sufficient conditions in order that all the algebraic limit cycles are hyperbolic. We also provide lower bounds for N .

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1. Introduction and statement of the main results

A (planar) polynomial differential system is a system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \quad \frac{dy}{dt} = \dot{y} = Q(x, y), \quad (1)$$

where P and Q are polynomials in the variables x and y . In this work the dependent variables x and y , the independent variable t , and the coefficients of the polynomials P and Q are all real be-

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