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Planar Vector Fields with a Given Set of Orbits

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Abstract We determine all the $\mathcal{C}^1$ planar vector fields with a given set of orbits of the form $y - y(x) = 0$ satisfying convenient assumptions. The case when these orbits are branches of an algebraic curve is also study. We show that if a quadratic vector field admits a unique irreducible invariant algebraic curve $g(x, y) = \sum_{j=0}^{S} a_j(x) y^{S-j} = 0$ with S branches with respect to the variable y, then the degree of the polynomial g is at most $4S$.

Keywords Invariant curve · Singular algebraic curve · Branches · Quadratic vector fields · Orthogonal polynomial · Orbits

Mathematics Subject Classification (2000) Primary 34C05 · 34A34 · 34C14

1 Introduction and Statement of the Main Results

By definition an autonomous complex planar differential system is a system of the form

$$\dot{\mathbf{x}} = \mathcal{X}(\mathbf{x}), \quad \mathbf{x} = (x, y) \in \mathrm{D} \subseteq \mathbb{C}^2, \tag{1}$$

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