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Invariant Parallels, Invariant Meridians and Limit Cycles of Polynomial Vector Fields on Some 2-Dimensional Algebraic Tori in $\mathbb{R}^3$

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Abstract We consider the polynomial vector fields of arbitrary degree in $\mathbb{R}^3$ having the 2-dimensional algebraic torus

$$\mathbb{T}^2(l, m, n) = \{(x, y, z) \in \mathbb{R}^3 : (x^{2l} + y^{2m} - r^2)^2 + z^{2n} - 1 = 0\},$$

where l, m, and n positive integers, and $r \in (1, \infty)$, invariant by their flow. We study the possible configurations of invariant meridians and parallels that these vector fields can exhibit on $\mathbb{T}^2(l, m, n)$. Furthermore, we analyze when these invariant meridians or parallels are limit cycles.

Keywords Polynomial vector field · Invariant parallel · Invariant meridian · Limit cycle · Periodic orbit · 2-Dimensional torus

Mathematics Subject Classification (2010) 34C05 · 37C10 · 37C27

1 Introduction and Statement of the Main Results

In 1878 Darboux published two works, [5] and [6], about polynomial vector fields or equivalently autonomous polynomial differential equations on $\mathbb{R}^n$ or $\mathbb{C}^n$. There, he showed that if a polynomial vector field has a sufficient number of invariant algebraic hypersurfaces then it has a first integral. If we have a polynomial vector field in $\mathbb{R}^n$ or $\mathbb{C}^n$ with a first integral, then we can reduces its study in one dimension; of course, in the planar case we can describe completely its phase portrait, which is the main goal of the qualitative theory of differential equations.

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