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Limit cycles bifurcating from isochronous surfaces of revolution in $\mathbb{R}^3$ ☆

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ABSTRACT

In this paper we study the number of limit cycles bifurcating from isochronous surfaces of revolution contained in $\mathbb{R}^3$, when we consider polynomial perturbations of arbitrary degree. The method for studying these limit cycles is based on the averaging theory and on the properties of Chebyshev systems. We present a new result on averaging theory and generalizations of some classical Chebyshev systems which allow us to obtain the main results.

1. Introduction

Consider a differential system

$$\dot{\mathbf{x}} = X_0(\mathbf{x}) + \varepsilon X(\mathbf{x}), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^3$, $X_0, X : \mathbb{R}^3 \to \mathbb{R}^3$ are vector fields and ε is a real small parameter; the dot denotes the derivative with respect to the time. If we suppose that $(1)_{\varepsilon=0}$ has an *isochronous invariant surface* $\mathcal{S} \subset \mathbb{R}^3$, that is, $\mathcal{S}$ is foliated by periodic orbits with the same period, then natural questions are: For $\varepsilon \neq 0$ sufficiently small does the differential system (1) possess limit cycles emerging from the periodic orbits of $\mathcal{S}$? How to compute them? How many? These questions are analogous to the following about planar differential systems: How many limit cycles emerge under a perturbation from a planar center? In this last case many results has been obtained (see for example [2] and the references therein). Recall that a *limit cycle* of a differential system is a periodic orbit which is isolated in the set of all periodic orbits of the system.

A tool for studying these kind of problems is the averaging theory. For instance, perturbations of isochronous sets of periodic orbits as planes, cylinders and tori in $\mathbb{R}^3$ has been studied, see [4–6]. For a general introduction to this theory see [9] and [11].

In this paper we consider differential systems $(1)_{\varepsilon=0}$ in $\mathbb{R}^3$ which contain an isolated isochronous invariant revolution surface of the form

$$\mathcal{S}_F = \left\{(x, y, z) \in \mathbb{R}^3 \;\middle|\; F(x, y, z) = x^2 + y^2 - f(z) = 0\right\}, \tag{2}$$

where $f(z) > 0$ in a nonempty open subset $\mathcal{U}_f$ of $\mathbb{R}$. Mainly we consider the quadratic case, that is, when X_0 is quadratic vector field and $f(z)$ is a polynomial of degree at most 2. The set of all these $\mathcal{S}_F$ contains the main quadratic surfaces of $\mathbb{R}^3$: the sphere $\{x^2 + y^2 + z^2 - 1 = 0\}$, the cylinder $\{x^2 + y^2 - 1 = 0\}$, the hyperboloid of one sheet $\{x^2 + y^2 - z^2 - 1 = 0\}$, the hyperboloid of two sheets $\{x^2 + y^2 - z^2 + 1 = 0\}$, the cone $\{x^2 + y^2 - z^2 = 0\}$ and the paraboloid $\{x^2 + y^2 - z = 0\}$.

☆ Limit cycles bifurcating from isochronous surfaces.

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