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Limit Cycles Bifurcating from a 2-Dimensional Isochronous Torus in $\mathbb{R}^3$

Jaume Llibre, Salomón Rebollo-Perdomo, Joan Torregrosa

Departament de Matemàtiques
Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, SPAIN
e-mails: jllibre@mat.uab.cat, srebollo@mat.uab.cat, torre@mat.uab.cat

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Abstract

In this paper we illustrate the explicit implementation of a method for computing limit cycles which bifurcate from a 2-dimensional isochronous set contained in $\mathbb{R}^3$, when we perturb it inside a class of differential systems. This method is based in the averaging theory. As far as we know all applications of this method have been made perturbing non-compact surfaces, as for instance a plane or a cylinder in $\mathbb{R}^3$. Here we consider polynomial perturbations of degree d of an isochronous torus. We prove that, up to first order in the perturbation, at most $2(d+1)$ limit cycles can bifurcate from a such torus and that there exist polynomial perturbations of degree d of the torus such that exactly ν limit cycles bifurcate from such a torus for every $\nu \in \{2, 4, \ldots, 2(d+1)\}$.

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1 Introduction

Let $\mathbb{T}$ be a 2-dimensional torus of an integrable Hamiltonian system of two degrees of freedom contained in a 3-dimensional energy level and foliated by periodic orbits. Under the assumptions of the Poincaré–Birkhoff Theorem we can study the periodic orbits of $\mathbb{T}$ which persist when we perturb

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