QUADRATIC SYSTEMS WITH AN INVARIANT CONIC HAVING DARBOUX INVARIANTS

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Abstract. The complete characterization of the phase portraits of real planar quadratic vector fields is very far to be completed. As this attempt is not possible in the whole class due to the large number of parameters (twelve, but, after affine transformations and time rescaling, we arrive at families with five parameters, which is still a big number of parameters), many subclasses have been considered and studied. In this paper we complete the characterization of the global phase portraits in the Poincaré disc of all planar quadratic polynomial differential systems having an invariant conic and a Darboux invariant, constructed using only the invariant conic.

1. Introduction

Denote by $\mathbb{R}[x, y]$ the ring of the real polynomials in the variables x and y, and consider the differential system in $\mathbb{R}^2$ given by

$$\begin{aligned} \dot{x} &= P(x, y), \\ \dot{y} &= Q(x, y), \end{aligned} \tag{1}$$

where $P, Q \in \mathbb{R}[x, y]$. In (1) the dot denotes derivative with respect to the *time* t and, we define the *degree* of system (1) as $m = \max\{\deg P, \deg Q\}$.

A *quadratic system* is a *quadratic polynomial differential system* as (1) for which $m = 2$. The quadratic systems appear in the modeling of many natural phenomena described in different branches of the sciences, and in biological and physical applications. Of course, the quadratic systems became a matter of interest for the mathematicians because after the linear differential systems they are the easiest polynomial differential systems. More than one thousand of papers have been published about quadratic systems. See for instance [8, 9] for a bibliographical survey.

Because we want to study quadratic systems, *in this paper we always assume that the polynomials P and Q are coprime,* otherwise system (1) can be reduced to a linear or constant system by a rescaling of the time variable.

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