On the periodic orbits and the integrability of the regularized Hill lunar problem

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The classical Hill's problem is a simplified version of the restricted three-body problem where the distance of the two massive bodies (say, primary for the largest one and secondary for the smallest one) is made infinity through the use of Hill's variables. The Levi-Civita regularization takes the Hamiltonian of the Hill lunar problem into the form of two uncoupled harmonic oscillators perturbed by the Coriolis force and the Sun action, polynomials of degree 4 and 6, respectively. In this paper, we study periodic orbits of the planar Hill problem using the averaging theory. Moreover, we provide information about the C^1 integrability or non-integrability of the regularized Hill lunar problem. © *2011 American Institute of Physics*. [doi:10.1063/1.3618280]

I. INTRODUCTION AND STATEMENT OF THE MAIN RESULTS

In this paper, we study periodic orbits of the Hill lunar problem and its C^1 non-integrability. The Hill lunar problem is a limit version of the restricted three-body problem, it is a model originally based on the Moon's motion under the joint action of the Earth and the Sun.[4] In the rotating frame the Hamiltonian of the Hill lunar problem is

$$H_{\mathrm{Hill}}(\mathbf{x}) = \frac{1}{2}(x_3^2 + x_4^2) + x_2x_3 - x_1x_4 - \frac{1}{\sqrt{x_1^2 + x_2^2}} - x_1^2 + \frac{1}{2}x_2^2, \tag{1.1}$$

where $\mathbf{x} = (x_1, x_2, x_3, x_4)$. For more details on this Hamiltonian, see Ref. 13.

To avoid the difficulties due to the collision is performed the Levi-Civita regularization as follows. We do the change of variables in the positions given by

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \hat{x}_1 & -\hat{x}_2 \\ \hat{x}_2 & \hat{x}_1 \end{pmatrix} \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix}, \tag{1.2}$$

and the induced change in the conjugate momenta is

$$\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \frac{2}{\hat{r}^2} \begin{pmatrix} \hat{x}_1 & -\hat{x}_2 \\ \hat{x}_2 & \hat{x}_1 \end{pmatrix} \begin{pmatrix} \hat{x}_3 \\ \hat{x}_4 \end{pmatrix}, \tag{1.3}$$

where $\hat{r}^2 = \hat{x}_1^2 + \hat{x}_2^2 = r = \sqrt{x_1^2 + x_2^2}$. To complete the regularization procedure it is necessary to rescale the time doing $d\tau = \dfrac{4dt}{(\hat{x}_1^2 + \hat{x}_2^2)}$. Applying these changes of variables in (1.1), and considering the Hamiltonian $\hat{H}(\hat{\mathbf{x}}) = \dfrac{r}{4}(H_{\mathrm{Hill}}(\mathbf{x}(\hat{\mathbf{x}})) + p_0)$, after omitting the hat of the variables, we get that the

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