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ON THE PERIODIC SOLUTIONS OF A CLASS OF DUFFING DIFFERENTIAL EQUATIONS

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To Professor Jean Mawhin, whose articles are always a pleasure to read

Abstract. In this work we study the periodic solutions, their stability and bifurcation for the class of Duffing differential equation $x'' + \varepsilon C x' + \varepsilon^2 A(t)x + b(t)x^3 = \varepsilon^3 \Lambda h(t)$, where $C > 0$, $\varepsilon > 0$ and Λ are real parameter, $A(t)$, $b(t)$ and $h(t)$ are continuous T–periodic functions and ε is sufficiently small. Our results are proved using the averaging method of first order.

1. **Introduction and statement of the main result.** Classes of Duffing differential equations have been investigated by many authors. They are interested in the existence of periodic solutions, in their multiplicity, stability, etc. See for example the works [1, 2, 3, 5] and specially the survey of J. Mawhin [4].

In this paper we deal with a subclass of Duffing differential equations of the form

$$x'' + cx' + a(t)x + b(t)x^3 = \lambda h(t), \tag{1}$$

where $c > 0$ is a constant, λ is a real parameter, $a(t)$, $b(t)$ and $h(t)$ are continuous T–periodic functions. These differential equations were studied by Chen and Li in [2]. In their work they consider the following additional conditions: $b(t) > 0$, $h(t) > 0$ and $a(t)$ satisfies

$$a(t) \leq \frac{\pi^2}{T^2} + \frac{c^2}{4}, \quad \text{and} \quad a_0 = \frac{1}{T}\int_0^T a(t)dt > 0. \tag{2}$$

Under these conditions they analyse the stability and the exact multiplicity of the T–periodic solutions of differential equation (1). On the other hand, Chen and Li in [1] also studied the Duffing differential equation (3) in a very particular case, i. e. $a(t) = a > 0$, $b(t) = 1$ and $c > 0$, a, c constants. They obtain sharp bounds for $h(t)$ in such away that (3) has exactly three T-periodic solutions.

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