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On the periodic orbits of the third-order differential equation $x''' - \mu x'' + x' - \mu x = \varepsilon F(x, x', x'')$

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ABSTRACT

In this paper we study the periodic orbits of the third-order differential equation $x''' - \mu x'' + x' - \mu x = \varepsilon F(x, x', x'')$, where ε is a small parameter and the function F is of class $\mathcal{C}^2$.

1. Introduction and statement of the main results

In the qualitative theory of differential equations one of the main problems is the study of their periodic orbits, existence, number and stability. A *limit cycle* of a differential equation is a periodic orbit isolated in the set of all periodic orbits of the differential equation.

In this paper we deal with the third-order differential equation

$$x''' - \mu x'' + x' - \mu x = \varepsilon F(x, x', x''). \tag{1}$$

Here the variables x and t, and the parameters μ and ε are real; moreover ε is a small real parameter and the function $F : \Omega \to \mathbb{R}$ is of class $\mathcal{C}^2$. Here Ω is an open subset of $\mathbb{R}^3$. The prime denotes the derivative with respect to an independent variable t. The objective is to study the periodic solutions of this differential equation.

There are many papers studying the periodic orbits of third-order differential equations. In particular, our class of equations (1) is not far from the ones studied in [1–5]. But our main tool for studying the periodic orbits of Eq. (1) is completely different to the tools of the mentioned papers. We shall use the *averaging theory*, more precisely Theorems 3 and 4 of the Appendix. Many of the papers dealing with the periodic orbits of third-order differential equations use Schauder's or Leray–Schauder's fixed point theorem, see for instance [6,7], or the nonlocal reduction method see [8], and others [9]. The non-autonomous case of the differential equation (1) was studied in [10] with $\mu \neq 0$. As in [10], our main tool for studying the periodic orbits of Eq. (1), was the averaging theory. But in [10] they only need to use Theorem 3, and here we shall use Theorem 3 when $\mu \neq 0$ and Theorem 4 when $\mu = 0$.

We recall that a *simple zero* r_0^* of a function $\mathcal{F}(r_0)$ is defined by $\mathcal{F}(r_0^*) = 0$ and $(d\mathcal{F}(r_0^*)/dr_0) \neq 0$.

The main results on the periodic solutions of the third-order differential equation (1) are the following two theorems.

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