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ON A CLASS OF CUBIC PLANAR POLYNOMIAL FOLIATIONS

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Abstract. Phase portraits of quadratic planar polynomial foliations are classified. Recently several papers deal with the phase portraits of cubic polynomial foliations. In this paper we determine all phase portraits of cubic planar polynomial foliations defined by all vector fields having the first component equal to a non–zero constant and the second one equal to an arbitrary cubic polynomial.

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1 Introduction

Let P and Q be real polynomials in the variables x and y of degrees m and n respectively, with $m \leq n$. Then we say that $X = (P, Q) : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ is a *polynomial vector field of degree n*.

We say that X is a *planar polynomial foliation of degree n* if $P(x, y)$ and $Q(x, y)$ have no common real zeros. From the Poincaré–Bendixson Theorem it is clear that any orbit of a planar polynomial foliation starts and ends at infinity, and consequently it divides the plane into two unbounded connected components. The orbits of a foliation are called the *leaves* of the foliation. We say that two planar polynomial foliations are *topologically equivalent* if there is a homeomorphism of $\mathbf{R}^2$ that carries the leaves of one foliation to those of the other, preserving or reversing simultaneously the sense of all leaves or orbits.

It is well–known that the topological classification of the foliations in the plane depends only on the number of the inseparable leaves and the way they are distributed in the plane (see, for instance [13] or [15]). Two leaves L_1 and L_2 are said to be *inseparable* if for any arcs T_1 and T_2 transversal to L_1 and