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J. Differential Equations 198 (2004) 374-380

Journal of Differential Equations

http://www.elsevier.com/locate/jde

Configurations of limit cycles and planar polynomial vector fields $\stackrel{\checkmark}{\rightarrowtail}$

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Received April 4, 2003; revised July 17, 2003

Abstract

We show that every finite configuration of disjoint simple closed curves of the plane is topologically realizable as the set of limit cycles of a polynomial vector field. Moreover, the realization can be made by algebraic limit cycles, and we provide an explicit polynomial vector field exhibiting any given finite configuration of limit cycles. © 2003 Elsevier Inc. All rights reserved.

MSC: primary 58F21; secondary 34C05; 58F14

Keywords: Limit cycles; Quadratic vector fields

1. Introduction

In 1900 Hilbert [6] in the second part of its 16th problem proposed to find an estimation of the uniform upper bound for the number of limit cycles of all polynomial vector fields of a given degree, and also to study their distribution or configuration in the plane. This has been one of the main problems in the qualitative theory of planar differential equations in the 20th century. The contributions of Bamon [3] for the particular case of quadratic vector fields, and mainly of Écalle [4]

 $^{^{*}}$ J.L. is partially supported by a MCYT and FEDER Grant BFM2002–04236–C02–02 and by a CIRIT Grant 2001SGR 00173; and G.R. is partially supported by a MCYT and FEDER number BFM2002–03161.

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