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On the periodic orbits of Hamiltonian systems

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We show how to apply to Hamiltonian differential systems recent results for studying the periodic orbits of a differential system using the averaging theory. We have chosen two classical integrable Hamiltonian systems, one with the Hooke potential and the other with the Kepler potential, and we study the periodic orbits which bifurcate from the periodic orbits of these integrable systems, first perturbing the Hooke Hamiltonian with a nonautonomous potential, and second perturbing the Kepler problem with an autonomous potential. © *2010 American Institute of Physics.* [doi:10.1063/1.3387343]

I. INTRODUCTION

In this paper we study the planar motion of a particle of unitary mass under the action of a central force with Hamiltonian given by

$$H_0(x,y,p_x,p_y) = \tfrac{1}{2}(p_x^2 + p_y^2) + V_0(\sqrt{x^2+y^2}),$$

perturbed by the Hamiltonian

$$H(x,y,p_x,p_y,t) = H_0(x,y,p_x,p_y) + \varepsilon V(t,x,y), \tag{1}$$

where ε is a small parameter and $V(t,x,y)$ is a perturbation of the potential eventually depending on the time t.

We consider a central force derived from a potential of the form

$$V_0(\sqrt{x^2+y^2}) = \pm (x^2+y^2)^{\alpha/2}. \tag{2}$$

The Hamilton equations associated to Hamiltonian (1) are

$$\dot{x} = p_x,$$

$$\dot{y} = p_y,$$

$$\dot{p}_x = -\partial(V_0(\sqrt{x^2+y^2}) + \varepsilon V(t,x,y))/\partial x,$$

$$\dot{p}_y = -\partial(V_0(\sqrt{x^2+y^2}) + \varepsilon V(t,x,y))/\partial y, \tag{3}$$

where the dot denotes derivative with respect to the time t. We shall apply the averaging theory for studying the periodic orbits of the Hamiltonian system (3).

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