DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS SERIES B
Volume **19**, Number **4**, June **2014**

doi:10.3934/dcdsb.2014.19.1129

pp. **1129–1136**

ON THE LIMIT CYCLES OF THE FLOQUET DIFFERENTIAL EQUATION

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(Communicated by Susanna Terracini)

Abstract. We provide sufficient conditions for the existence of limit cycles for the Floquet differential equations $\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + \varepsilon(B(t)\mathbf{x}(t) + b(t))$, where $\mathbf{x}(t)$ and $b(t)$ are column vectors of length n, A and $B(t)$ are $n \times n$ matrices, the components of $b(t)$ and $B(t)$ are T–periodic functions, the differential equation $\dot{\mathbf{x}}(t) = A\mathbf{x}(t)$ has a plane filled with T–periodic orbits, and ε is a small parameter. The proof of this result is based on averaging theory but only uses linear algebra.

1. Introduction.

The linear first order differential equation

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) + b(t), \tag{1}$$

where $\mathbf{x}(t)$ and $b(t)$ are column vectors of length n, $A(t)$ is an $n \times n$ matrix, and $A(t)$ and $b(t)$ are periodic with period T or simply *T–periodic* (i.e. $A(t+T) = A(t)$ and $b(t+T) = b(t)$ for all $t \in \mathbb{R}$), is called a *Floquet differential equation*. For more details on the Floquet differential equations see [3]. These differential equations have been studied intensively and have many applications, see for examples the papers of Trench [11, 12] and the references quoted therein. As far as we know there are no general results on the existence or non–existence of limit cycles for the Floquet differential equations. The objective of this paper is to provide sufficient conditions for the existence of limit cycles for a subclass of Floquet differential equations.

A *limit cycle* of the differential equation (1) is a periodic orbit isolated in the set of all periodic orbits of the differential equation (1). To obtain analytically limit cycles of a differential equation is in general a very difficult problem, many times impossible. If the averaging theory can be applied to the differential equation (1), then it reduces this difficult problem to find the zeros of a nonlinear function. It is known that in general the averaging theory for finding limit cycles does not provide all the limit cycles of the differential equation.

2010 *Mathematics Subject Classification.* Primary: 58F15, 58F17; Secondary: 53C35.

Key words and phrases. Floquet differential equation, periodic solution, limit cycle, averaging theory.