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Minimal periods of holomorphic maps on complex tori

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We study the set of minimal periods of holomorphic self-maps of one- and two-dimensional complex tori. In particular, we characterize when the set of minimal periods of such maps is finite. In fact, we have an algorithm for doing this characterization for holomorphic self-maps of an arbitrary dimensional complex torus.

Keywords: set of minimal periods; periodic points; holomorphic maps on complex tori

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1. Introduction and statement of the main results

One of the classical invariants in the study of dynamical properties of a map f is the set of minimal periods $\mathrm{Per}(f)$. This set in general is not stable, i.e. it changes if we perturb the map. In particular, it is not preserved by a homotopy of the map. It is difficult to analyse the set $\mathrm{Per}(f)$ using tools from algebraic topology. To avoid this difficulty, many authors studied the set of homotopy minimal periods, i.e. minimal periods that are preserved by any homotopy (see for instance [1,7,10,12,14], and [8, Chap. VI], for an exposition of known results).

On the other hand, it is known that holomorphic maps of compact complex manifolds have many periodic points and large sets of minimal periods (see [3], [5] and [13]). Therefore, a natural question is: 'which minimal periods of a holomorphic map f are preserved by a holomorphic homotopy of f, and which of them are preserved by any continuous deformation of f?', see the appendix for definitions and more details.

In this paper, we consider this question for holomorphic self-maps of one- and two-dimensional complex tori, i.e. topologically two- and four-dimensional real tori. The answer is complete when the set of these minimal periods is finite. The same question for holomorphic self-maps of the one-dimensional complex torus was partially studied in [13].

Let $\mathbb{T}^r$ be the r-dimensional torus of complex dimension r; i.e. $\mathbb{T}^r = \mathbb{C}^r/\Lambda$ with Λ a co-compact lattice. Of course as usual $\mathbb{C}$ denotes the set of complex numbers. Let $f : \mathbb{T}^r \to \mathbb{T}^r$ be a holomorphic map. We denote by A_f the matrix of the induced homology homomorphism $f_* : H_1(\mathbb{T}^r) \to H_1(\mathbb{T}^r)$ on the first homological group of $\mathbb{T}^r$, which is a $2r \times 2r$ integer matrix. It is well known that in an appropriate basis $f = A_f$, see [4,13], for more details.

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