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ON THE NUMBER OF LIMIT CYCLES FOR DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS IN $\mathbb{R}^{2n}$ WITH TWO ZONES

JAUME LLIBRE
Departament de Matemàtiques,
Universitat Autònoma de Barcelona, 08193 Bellaterra,
Barcelona, Catalonia, Spain
jllibre@mat.uab.cat

FENG RONG
Department of Mathematics, Shanghai Jiao Tong University,
800 Dong Chuan Road, Shanghai 200240, P. R. China
frong@sjtu.edu.cn

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We study the number of limit cycles of the discontinuous piecewise linear differential systems in $\mathbb{R}^{2n}$ with two zones separated by a hyperplane. Our main result shows that at most $(8n-6)^{n-1}$ limit cycles can bifurcate up to first-order expansion of the displacement function with respect to a small parameter. For proving this result, we use the averaging theory in a form where the differentiability of the system is not necessary.

Keywords: Limit cycles; averaging method; discontinuous piecewise linear differential systems.

1. Introduction and Statement of the Main Result

For a given differential system, a *limit cycle* is a periodic orbit isolated in the set of all its periodic orbits. Within the qualitative theory of differential systems, the study of their limit cycles is one of the main topics.

Many questions are considered on the limit cycles of the differential systems in $\mathbb{R}^2$. Thus, one of the main lines of research for such systems is the study of how many limit cycles emerge from the periodic orbits of a center when we perturb it inside a given class of differential equations, see for example, [Christopher & Li, 2007] and the references therein. More precisely, the problem is to consider the planar linear differential center

$$\dot{x} = -y, \quad \dot{y} = x$$

and perturb it

$$\dot{x} = -y + \varepsilon f(x,y), \quad \dot{y} = x + \varepsilon g(x,y),$$

inside a given class of differential equations for studying the limit cycles which bifurcate from the periodic orbits of the linear center. Of course, ε is a small parameter. Here, our main concern is to bring this problem to higher dimension when the perturbation is discontinuous and piecewise linear.

In [Lum & Chua, 1990] the authors conjectured that a continuous piecewise linear differential system in the plane with two zones separated by a straight line has at most one limit cycle. This conjecture was proved by Freire *et al.* [1998]. The number of limit cycles for continuous piecewise linear differential system with three zones separated by two parallel hyperplanes in $\mathbb{R}^{2n}$ has been studied in [Cardin *et al.*, 2012].