Minimal set of periods for continuous self–maps of a bouquet of circles

Jaume Llibre[1] and Ana Sá[2]

[1] Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain. e–mail: jllibre@mat.uab.es

[2] Departamento de Matemática, Faculdade de Ciências e Tecnologia, Universidade Nova de Lisboa, 2829-516 Monte de Caparica, Portugal. e–mail: ams@fct.unl.pt

Abstract

Let G_k be a bouquet of circles; i.e. the quotient space of the interval $[0, k]$ obtained by identifying all points of integer coordinates to a single point, called the branching point of G_k. Thus, G_1 is the circle, G_2 is the eight space and G_3 is the trefoil. Let $f : G_k \to G_k$ a continuous map such that for $k > 1$ the branching point is fixed.

If Per(f) denotes the set of periods of f, the minimal set of periods of f, denoted by MPer(f), is defined as $\bigcap_{g\simeq f}$ Per(g) where $g : G_k \to G_k$ is homological to f.

The sets MPer(f) are well–known for circle maps. Here, we classify all the sets MPer(f) for self–maps of the eight space and the trefoil.

1 Introduction and statement of the results

In dynamical systems it is often the case that topological information can be used to study qualitative or quantitative properties of the system. This work deals with the problem of determining the set of periods of periodic orbits of a map given the homology class of the map.

A *finite graph* (simply a *graph*) G is a topological space formed by a finite set of points V (points of V are called *vertices*) and a finite set of open arcs (called *edges*) in such a way that each open arc is attached by its endpoints to vertices. An open arc is a subset of G homeomorphic to the open interval $(0, 1)$. Note that a finite graph is compact, since it is the union of a finite number of compact subsets (the closed edges and the vertices). Notice that a closed edge is homeomorphic either to the closed interval [0,1], or to the circle. It may be either connected or disconnected, and it may have isolated vertices.

The *valence* of a vertex is the number of edges with the vertex as an endpoint (where the closed edges homeomorphic to a circle are counted twice). We say that a graph is *proper* if the valence of all its vertices is distinct from two. *In all this work the graphs will be always proper*. The vertices with valence 1 of a connected graph are *endpoints* of the graph and the vertices with valence larger than 2 are *branching points*.

Suppose that $f : G \to G$ is a continuous map, in what follows a *graph map*. A *fixed point* of f is a point x in G such that $f(x) = x$. We will call x a *periodic point of period* n if x is a fixed point of f^n but it is not fixed by any f^k for $1 \le k < n$. We denote by Per(f) the set of natural numbers corresponding to periods of periodic points of f.