ON THE ANALYTIC INTEGRABILITY OF THE 5–DIMENSIONAL LORENZ SYSTEM FOR THE GRAVITY–WAVE ACTIVITY

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ABSTRACT. The 5-dimensional Lorenz system for the coupled Rosby and gravity waves has exactly two independent analytic first integrals.

1. INTRODUCTION

E.N. Lorenz constructed in [4] the following 5–dimensional differential system in $\mathbb{R}^5$

$$\begin{aligned} dU/dT &= -VW + b\,VZ, \\ dV/dT &= UW - b\,UZ, \\ dW/dT &= -UV, \\ dX/dT &= -Z, \\ dZ/dT &= b\,UV + X, \end{aligned} \qquad (L)$$

where $b \in \mathbb{R}$ is a parameter, describing coupled Rosby waves and gravity waves.

He was mainly interested in its slow manifolds. Here our interest will be in studying its analytic integrability, i.e. *what is the maximal number of independent analytic first integrals that the system* (L) *can exhibit*? This question has been considered for many other relevant differential systems and other classes of first integrals not necessarily analytic, see for instance [2], [5], [6], ...

Let U be an open subset of $\mathbb{R}^5$ invariant by the flow of the differential system (L), i.e. if a solution of system (L) has a point in U then all the points of this solution are contained in U. A *first integral* of the differential system (L) in U is a non–constant function $H = H(U, V, W, X, Z) : U \to \mathbb{R}$ satisfying that it is constant on every solution of system (L) contained in U. In other words, H is a first

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