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The index of singularities of vector fields and finite jets

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ABSTRACT

We describe when the index of a singularity of a smooth vector field is determined by a finite jet at the singularity. We also give some criteria to determine some terms from the formal series expansion of the vector field at the singularity which determine the index.

1. Introduction and statement of results

There exists a large literature regarding whether some finite jet of a smooth vector field determines the phase portrait near a singularity up to C^0 conjugacy (or even smooth conjugacies), starting with Hartman–Grobman Theorem for 1-jets (Sternberg for higher regularity), continuing with results by Takens, Dumortier and others for higher order jets (see [4,5,9]). There is also some work in determining whether some terms in the formal series expansion at a singularity determine the local phase portrait, using homogeneous or quasi-homogeneous components, or more general the Newton diagram and the principal part (see [2,6,7]). The aim of this paper is to obtain similar results, when instead of determining the phase portrait, we only want to determine the index of the singularity.

Let $\mathcal{V}^n$ be the space of C^∞ vector fields on $\mathbb{R}^n$ with a singularity (a zero of the vector field, or a fixed point for the corresponding flow) at the origin, and J^n_k the space of k-jets of (germs of) such vector fields, where $k \in \mathbb{N} \cup \{\infty\}$. Abusing notations, we denote by $\pi_k : \mathcal{V}^n$, $J^n_l \to J^n_k$ the natural projection from the respective spaces to the space of k-jets, for $l \geqslant k$. Let $\mathcal{J}$ be the ring of jets of (germs of) C^∞ maps from $\mathbb{R}^n$ to $\mathbb{R}$. Most of the following definitions and results can be extended to vector fields with enough regularity, or to finite jets, but for simplicity we restrict ourselves to the smooth case. Also we will see later that the concept of index stability is equivalent with the fact that the

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