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On the dynamics of a class of Kolmogorov systems

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ABSTRACT

We characterize the integrability and the non-existence of periodic orbits for the 2-dimensional Kolmogorov systems of the form

$$\dot{x} = x(P_n(x,y) + R_m(x,y)),$$
$$\dot{y} = y(Q_n(x,y) + R_m(x,y)),$$

where n and m are positive integers and P_n, Q_n and R_m are homogeneous polynomials of degree n, n and m, respectively.

1. Introduction and statement of the main results

The so called Kolmogorov systems are differential equations of the form

$$\dot{x}_i = x_i f_i(x_1, \ldots, x_n) \quad \text{for } i = 1, \ldots, n.$$

These systems appear in applications that the per unit of change $\dot{x}_i/x_i$ of the dependent variables $x_i = x_i(t)$ are given functions $f_i(x_1, \ldots, x_n)$ of these variables at any time. These systems are also called *Lotka–Volterra systems* because were started to be studied by them in [19] and in [23], respectively. Later on Kolmogorov came, giving some generalizations in [14] and then some authors denote them by Kolmogorov systems.

There are many natural phenomena which can be modeled by the Kolmogorov systems such as mathematical ecology and population dynamics [21], chemical reactions, plasma physics [15], hydrodynamics [3], economics, etc.

Starting with Volterra, mathematicians have been interested in Kolmogorov systems particularly in

- their integrability, i.e. when such differential systems have first integrals (see for instance [1,2,4–8,17,18,22]),or
- in their periodic orbits (see for example [9–11,13,16,20,24–26]).

See also the references quoted in those articles.

In this paper we are interested in studying the integrability and the periodic orbits of the 2-dimensional Kolmogorov systems of the form

$$\begin{aligned} \dot{x} &= x(P_n(x,y) + R_m(x,y)), \\ \dot{y} &= y(Q_n(x,y) + R_m(x,y)), \end{aligned} \qquad (1)$$

where n and m are positive integers and P_n, Q_n and R_m are homogeneous polynomials of degree n, n and m, respectively.

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