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On the limit cycles of the piecewise differential systems formed by a linear focus or center and a quadratic weak focus or center

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ABSTRACT

While the limit cycles of the discontinuous piecewise differential systems formed by two linear differential systems separated by one straight line have been studied intensively, and up to now there are examples of these systems with at most 3 limit cycles. There are almost no works studying the limit cycles of the discontinuous piecewise differential systems formed by one linear differential system and a quadratic polynomial differential system separated by one straight line.

In this paper using the averaging theory up to seven order we prove that the discontinuous piecewise differential systems formed by a linear focus or center and a quadratic weak focus or center separated by one straight line can have 8 limit cycles. More precisely, at every order of the averaging theory from order one to order seven we provide the maximum number of limit cycles that can be obtained using the averaging theory.

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1. Introduction and statement of the main results

Piecewise differential systems are provided as one of the most remarkable non-smooth dynamical systems and widely applied in various scientific domains of studies such as engineering, electronics, and physics [1,2,8,12,15,16,24,26,27]. Since the 1930s many books and papers study the piecewise differential systems, mainly due to their applications to mechanics and electrical circuits, see for instance [6,7,25,28]. The more studied piecewise differential systems are the continuous and discontinuous piecewise differential systems separated by a straight-line, see for instance [4,9–11,17–23].

A limit cycle is an isolated periodic orbit in the set of all periodic orbits of a differential system. Limit cycles play a main role in the qualitative theory of the differential systems, and also in the discontinuous piecewise differential systems. The singular point $p \in \mathbb{R}^2$ is a center of a planar differential system if there is a neighborhood U of p where all the orbits of $U \backslash \{p\}$ are periodic.

Our objective is to study the limit cycles which bifurcate from the periodic orbits of the linear differential center $\dot{x} = -y, \quad \dot{y} = x$, when we perturb this center by discontinuous piecewise differential systems separated by the straight line $y = 0$ and formed by linear differential focus or center

$$\dot{x} = \alpha x + \beta y + \gamma, \quad \dot{y} = -\beta x + \alpha y + \delta \tag{1}$$

defined in $y \geq 0$, and quadratic weak focus or center at the origin

$$\dot{x} = -y - bx^2 - cxy - dy^2, \quad \dot{y} = x + ax^2 + Axy - ay^2, \tag{2}$$

defined in $y \leq 0$. For more details on the quadratic weak focus or center see Lemma 8.14 of [5].

Our main result is the following theorem.

Theorem 1. *For $\varepsilon \neq 0$ sufficiently small the maximum number of limit cycles of the piecewise differential systems obtained perturbing the linear differential center $\dot{x} = -y$, $\dot{y} = x$ by the discontinuous piecewise differential system formed by systems (1) and (2) obtained using averaging theory up to seven order is eight.*

Theorem 1 is proved in Section 3. We note that in general to study analytically the limit cycles is a very difficult task, here we do this study using the new theory of averaging for discontinuos piecewise differential systems developed in [14], a summary of this theory is given in Section 2.

2. The averaging theory up to order 7 for computing limit cycles

In this section we present the basic results from the averaging theory for computing the periodic solutions of discontinuous piecewise differential systems that we shall need for proving the main results of this paper. This improvement of the classical averaging theory for computing limit cycles of planar discontinuous piecewise differential systems was developed in [14], a summary of this theory is given in below. We consider discontinous differential systems of the form

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